Numerical analysis of vertical stirred mills scale-up using discrete element method

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ABSTRACT

Vertical stirred mills (Vertimill[TM]) are being used in the regrinding circuits of several mineral processing plants. This type of mill is known to present lower specific energy consumption (kWh/t) when compared to its ball mill counterparts. One supposed source of ball mill inefficiency is not presented in vertical stirred mills, which is the waste of energy due to ball lifting, and this reflects into a more optimal collision energy spectrum provided by the grinding media to break particles. The outcome is that the vertical stirred mill imposes impacts of lower magnitudes and higher frequency of collisions. In this work, the energy spectrum of similar performance vertical stirred mill and ball mill were predicted using the discrete element method and predictions of breakage rates of the different size reduction machines using the UFRJ mechanistic mill model confirmed the higher efficiency of vertical stirred mills, as has been observed in industrial practice.

Keywords: Vertical stirred mills, Jar test, Scale-up, Simulation, Discrete element method.

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1. Introduction

In the technical literature, many authors have stated that the vertical stirred mill is more efficient than the ball mill regarding specific energy consumption to achieve a similar product, being the vertical stirred mill, typically between 30% and 50% more efficient than the ball mill (Vanderbeek, 1998; Jankovic, Valery, Clarke, 2006; Lichter & Davey, 2006; Rosa, Oliveira, Donda, 2014; Mazzinghy et al., 2015; Mazzinghy et al., 2017). In this way, the vertical stirred mill has been considered an attractive option in fine grinding in most new mining projects, mainly in the mineral regrinding applications. This type of mill presents lower specific energy consumption (kWh/t) when compared to ball mills and part of this advantage arises from looking into the collision energy spectra of grinding media. It has been stated, for instance, that the vertical stirred mill does not waste energy on lifting the balls, providing lower magnitude of impacts and increased frequency of collisions in the mill charge. In preliminary scale-up exercises, the energy consumption of vertical stirred mills is usually estimated by vertical stirred mill manufacturers through laboratory-scale tests with conventional tubular ball mills, usually named “jar tests” (Wills and Finch, 2016). Typically, the energy consumption in kWh/t obtained from the jar test is simply multiplied by a factor of about 0.65 (65% of a ball mill specific energy) to give the expected vertical stirred mill consumption (Mazzinghy et al., 2015) and this estimate used to scale-up to industrial vertical stirred mills. In practice, this methodology has been used in the initial design phase as a preliminary estimate for sizing vertical stirred mills, but normally followed by confirmatory pilot-scale testing (Mazzinghy et al., 2015).

The present work shows a comparison of collision energy spectra of mill charge on vertical stirred mills and ball mills, obtained using the EDEM software (EDEM, 2016). This information, along with published material breakage characterization data, is then used to predict the breakage rates of particles contained in different sizes, making it possible to verify the factor used to scale-up the specific mill power at the preliminary stage of design of vertical stirred mills.

2. Background

2.1 Discrete element method

Cundall & Strack (1979) were the first to detail all the steps necessary to describe mathematically the contacts between particles in order to show arrangements and particulate systems behaviors. Each particle is represented in three dimensions and is governed by mass, the radius and the moment of inertia. The normal and shear forces (tangential) according to Newton’s second law can be describe by the contact between particles elements. The ability to model comminution techniques has been key in many industries including mineral processing, cement, food processing, pigments and industrial minerals and pharmaceuticals (Cleary, Sinnott, Morrison, 2008) where size reduction is the main focus (Weerasekara et al., 2013). In mineral processing, grinding is a critical unit operation which is often performed in autogenous (AG), semi-autogenous (SAG), ball and stirred mills (Cleary, 2015). Over the last two decades, the discrete element method (DEM) has been used extensively as the basis for modelling different types of grinding devices as described in details at Weerasekara et al. (2013) and listed by Cleary (2015). The 2D and 3D description of charge motion was done by Mishra & Rajamani (1992, 1994), Powell et al. (2011) and Sinnott, Cleary, Morrison (2017), respectively, for ball mills. The performance of SAG mills was investigated by Rajamani and Mishra (1996), Powell, McBride, Govender (2003), Cleary & Morrison (2004), Morrison et al. (2006), Cleary, Sinnott, Morrison (2006a) and Carvalho and Tavares (2014). In turn, vertical stirred mills physics was described in terms of its energetic performance (Sinnott, Cleary, Morrison, 2006), charge motion and collisional structure (Cleary, Sinnott, Morrison, 2006b), media shape and properties (Sinnott, Cleary, Morrison, 2011a) and comparison between ball mill and vertical stirred mill (Morrison, Cleary, Sinnott, 2009). The inclusion of hydrodynamic effects of the slurry is presented by Sinnott, Cleary, Morrison (2011b) and Cleary (2015) towards the use of a Lagrangian, numerical method named Smoothed Particle Hydrodynamics (SPH) (Gingold & Monaghan, 1977; Hoover, 2006).

2.2 Mechanistic mill model

Several different approaches have been proposed to use information from collision energy spectra collected obtained from DEM simulations to predict the performance of tumbling mills (Tavares, 2017). One particularly attractive and detailed approach has been proposed by researchers from Universidade Federal do Rio de Janeiro, being called UFRJ mechanistic mill model (Tavares and Carvalho, 2009; Carvalho and Tavares, 2013; Rodriguez, 2016). This approach allows predicting the product size distribution and, from which, the linearized breakage rates, of particles of different sizes, taking into account the different breakage mechanisms, namely body and surface breakage, as well as damage accumulation, also accounting for the variability in particle strengths (Tavares, 2017). This model has been recently improved (Rodriguez, 2016) by accounting for the effect of the shear component of the collisions, previously not considered in the simulations (Tavares and Carvalho, 2009), with an improved energy-dependent surface breakage model and also by accounting for the possibility of unsuccessful collision events in the mill. Detailed descriptions of the model may be found elsewhere (Carvalho and Tavares, 2013; Rodriguez, 2016).

2.3 Mill power estimation

Power draw models for grinding mills have been developed by many authors. In the case of ball mills, a common approach is to use Rowland’s equation for mills smaller than 2.44 m in diameter (Rowland, 1986), which is expressed by:

$$P_{BM} = 6.3D^{0.3} sen\left(51 - 22\left(\frac{2.44 - D}{2.44}\right)\right)(3.2 - 3J)C_s\left(1 - \frac{0.1}{2^{(9-10C_s)}}\right)M_b \qquad (1)$$

where P_{BM} is the ball mill net power (W); D is the mill diameter inside liners (m), J is the fraction of mill volume loaded with balls, Cs is the fraction of critical speed and M_b is the mass of balls (kg).

A more recent equation is proposed by Stamboliadis *et al.* (2011), given by:

$$P_{BM} = 9.9\left(M_b + M_s\right)ND \qquad (2)$$

where M_s is the mass of material (kg); N is the rotational speed of the mill (Hz).

Equations for predictions of power draw in vertical stirred mills are not less numerous. Duffy (1994) developed an empirical equation to estimate the vertical stirred mill net power P_{VM} from five operations, being one in pilot-scale and four in industrial-scale, giving:

$$P_{VM} = 0{,}0743 L\omega\rho_c d^{\,0{,}111} D_s^{\,3{,}057} T^{\,0{,}572} \qquad (3)$$

where P_{VM} is the vertical stirred mill net power (kW); L is the balls bed height (m); ω is the screw angular speed (rpm); ρ_c is the mill charge effective density (t/m³); d is the mean balls size (mm); D_s is the screw diameter (m) and T is the number of screw turns.

The effective density of the mill charge can be calculated according to Equation 4.

$$\rho_c = \rho_b\left(1-\varepsilon\right) + \varepsilon\rho_s \qquad (4)$$

where ε is the fraction of empty space between the balls (dimensionless); ρ_b is the density of balls (t/m³) and ρ_s is the slurry density (t/m³).

An empirical equation was developed to predict the power of empty vertical stirred mill power $P_{no\text{-}load}$, giving:

$$P_{no-load} = 0{,}000134\omega W D_e^{0{,}57} \qquad (5)$$

$P_{no\text{-}load}$ is the empty vertical stirred mill power (kW) and W is the screw weight (kg).

Jankovic & Morrell (1997) developed an empirical equation using the power measured on 58 operations considering laboratory, pilot and industrial scales, given by:

$$P_{VM} = \frac{2{,}05\rho_c D^{\,1{,}96}\theta^{0{,}65}L^{0{,}98}d^{\,0{,}17}}{1000} \qquad (6)$$

where θ = peripheral screw speed (m/s).

The mill charge effective density was calculated according to Equation 7, proposed by Tüzün (1993):

$$\rho_c = (1-\varepsilon)(\rho_b - \rho_s) \qquad (7)$$

Nitta et al. (2006) developed an equation to estimate the vertical stirred mill gross power P_{VM*}, as shown in the equation:

$$P_{VM*} = 312 L^{0,8847} D^{2,232} 60 \omega^{1,232} d_{gap} \qquad (8)$$

where P_{VM*} is the vertical stirred mill gross power (kW); d_{gap} is the distance between the screw and the wall of vertical mill (m). The authors state that the error of this equation is around ± 10%.

3. Methodology

The discrete element method simulation in this work were performed using EDEM Academic platform (DEM Solutions, 2016), considering only the steel balls as the charge in the DEM environment. The contact model used was Hertz-Mindlin (no slip) approach (EDEM, 2016; Tsuji, Tanaka, Ishida, 1992; Di Renzo & Di Maio, 2004), in which the contact parameters between particles were calibrated for a laboratory-scale ball mill as described in Rodriguez (2016). A summary of the parameters used in the simulations is presented in Table 1.

Table 1. Material and contact parameters used in the DEM simulations

Variable	Value
Poisson’s Ratio	0.25
Shear Modulus (Pa)	$1.0e^{10}$
Density (kg/m^3)	7,800
Coefficient of Restitution	0.40
Coefficient of Static Friction	0.45
Coefficient of Rolling Friction	0.15

The data extraction was performed during 10 seconds of real life simulation after steady stated conditions were reached in the DEM simulations. The energy dissipation/loss registered for each ball-ball and ball-walls collisions were registered and then post-processing using an approach described elsewhere (Collinao et al, 2014). A common result from this post-processing technique is the collision energy spectrum for each possible pair of collision in the simulation, for instance, ball-liners, ball-ball, ball-screw. Thus, the power draw can be obtained by integrating the collision energy losses for a certain period.

In order to extend the performance comparison of both mill types, an additional analysis was made by coupling the collision energy spectra to the mechanistic modeling approach as developed by UFRJ research group (Universidade Federal do Rio de Janeiro), which has previously present good results when modeling ball mill and other comminution processes (Tavares & Carvalho, 2009; Carvalho & Tavares, 2013). The mechanistic modeling approach allowed predictions of the particle size distribution after certain period of time. Details on each simulation case are presented as follows. These simulations were all conducted considering 100% powder filling the voids left in the media charge.

3.1 Vertical stirred mill simulations

The simulations performed in the present work used a Vertimill VTM-1500 model as basis. This machine is currently being used in a new iron ore operation in Brazil.

In order to reduce the simulation time, and computational effort, while keeping good representative charge movement characteristics, a 1/10th scaled down version of the 3-D geometry of the VTM-1500 was built using CAD tools. The screw speed for the scaled down version was reduced based on database from authors and literature references (Menacho & Reyes, 1989, Duffy, 1994; Jankovic, 1999). Figure 1 shows the relation between screw speed and screw diameter for many vertical stirred mills sizes.

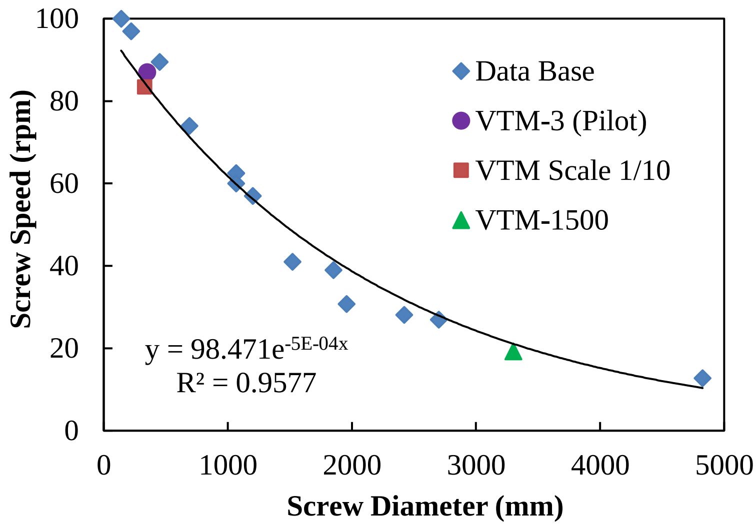

Figure 1. Relation between screw speed and screw diameter

The screw speed for the 1/10th scaled down version it is much closer to the VTM-3, normally used by Metso for pilot tests to confirm the Vertimill^{TM} sizing from jar tests, respectively 83 rpm and 87 rpm. Table 2 presents data for the VTM-1500 and the simulated vertical stirred mill in 1/10 scale.

Table 2. Dimensions and operation parameters of VTM-1500 and its scaled down version considered in the DEM simulations

Mill	VTM-1500	Scaled down
Scale	1/1	1/10
Screw Diameter (mm)	3300	330
Rotation Speed (rpm)	19	83

3.2 Ball mill simulations

The jar test normally used by Metso to scale-up Vertimill^{TM} was chosen as basis to DEM simulations of ball mills that allows the comparison to vertical stirred mills performance. In this case, both the mill used in the jar test and the vertical stirred mill simulated used the same ball size in the grinding charge, which as 17 mm. The mill used in the jar test is a steel cylinder with no lifters (Wills and Finch, 2016). Table 3 shows the dimensions and operations parameters of the jar test.

Table 3. Dimensions and operation parameters of the jar test

Diameter (mm)	203
Length (mm)	254
Ball Bed Porosity (%)	40
Ball Charge – *J* (%)	42
Powder Filling – *U* (%)	100
Critical Speed (%)	76

4. Results and discussion

A snapshot of DEM simulations is found in Figure 2, which displays the trajectory of each ball colored as function of their kinetic energy.

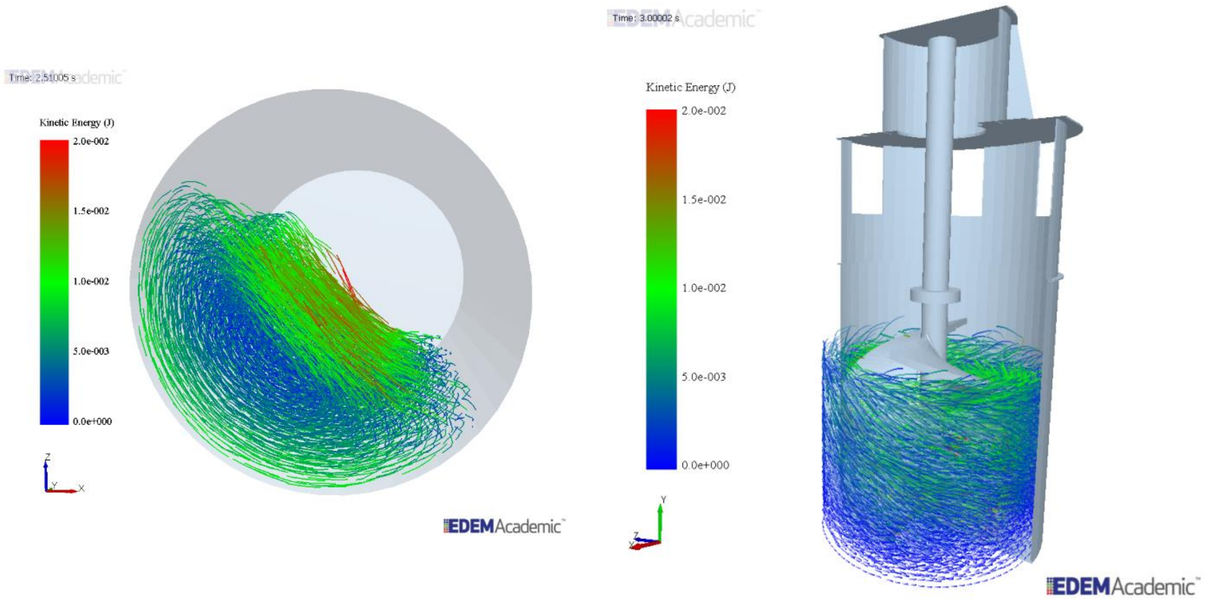

Figure 2. Streamlines showing charge motion for each case: jar test (left) and vertical stirred mill (right).

The DEM simulations of the jar test showed that collisions can reach energies of up to 10^{-2} J during the cascading movement. The calculated power draw for this mill was 0.049 kW, being this value close to the experimental values and the power estimated using empirical equations, as shown on Table 4 (Rowland, 1986; Stamboliadis et al., 2011).

Table 4. Mill power estimated by empirical equations and by DEM simulations

Mill	Model	Reference	Power (kW)
Ball Mill	Equation 1	Rowland (1986)	0.046
	Equation 2	Stamboliadis et al. (2011)	0.039
	DEM Simulation	-	0.049
Vertical Stirred Mill	Equation 3	Duffy (1994)	0.62
	Equation 6	Jankovic & Morrell (1997)	1.08
	Equation 8	Nitta et al. (2006)	0.64
	DEM Simulation	-	0.72

When comparing to the vertical stirred mill simulation, displayed in Figure 2 (right), it can be seen that kinetic energy levels of the balls is similar to those observed in jar test. However, the larger number of balls in the charge make up the power draw to 0.72 kW (Table 4). The estimated values of power from the DEM simulation of the ball mill matched well the predictions from the different empirical equations. The vertical stirred mill power obtained by Equation 8 from Nitta et al. (2006) is the gross power that was adjusted considering Equation 5 from Duffy (1994) for no-load power, obtaining at the end the net power, as estimated by Duffy (1994) and Jankovic & Morrell (1997) and comparable directed with power obtained from DEM simulations.

The vertical motion of the balls towards the upper region of the mill was observed in the central region near the stirrer, depicted by the red particles in Figure 3, while the descending motion was observed along the periphery of the vessel, illustrated by green particles.

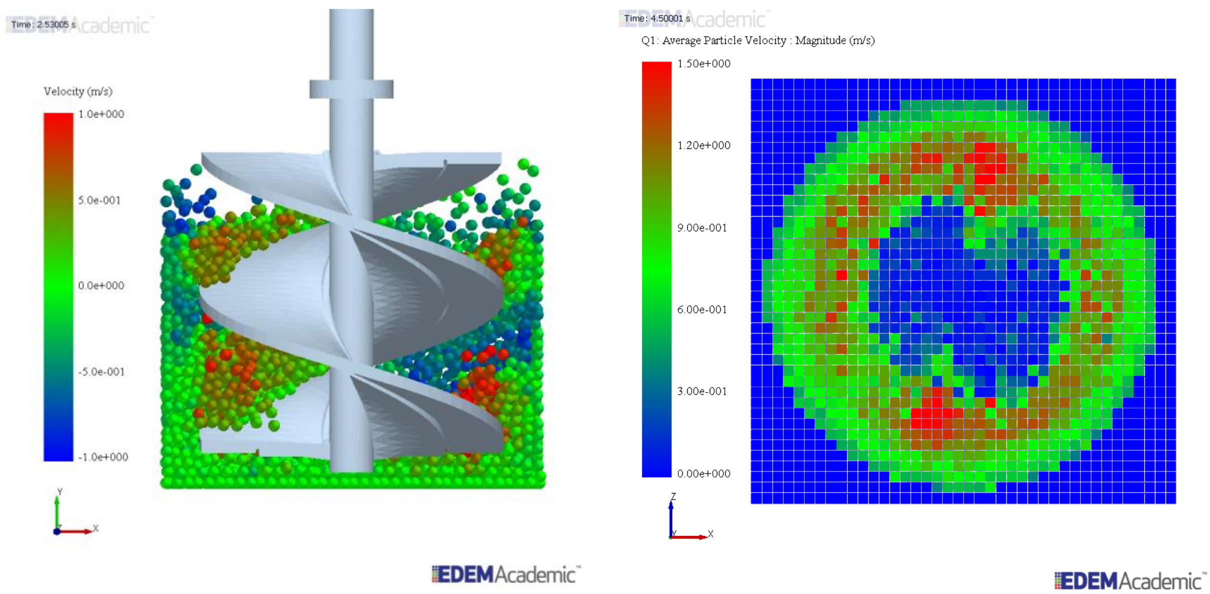

Figure 3. Sections showing the charge motion within the vertical stirred mill.

A comparison of the collision energy spectra for both mills shows that the highest energy loss in the jar test is around 10^{-2} J (Figure 4), while the vertical stirred mill shows a number of collisions that reach up to 1 J (Figure 5). It also noted the energy losses due to shear collisions are dominant in the vertical stirred mill while the normal collisions show higher values in the jar test.

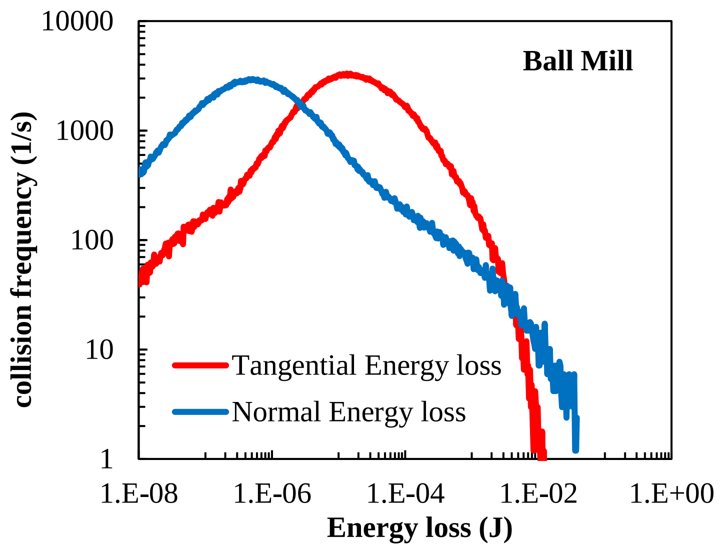

Figure 4. Energy and frequency of collisions for jar test

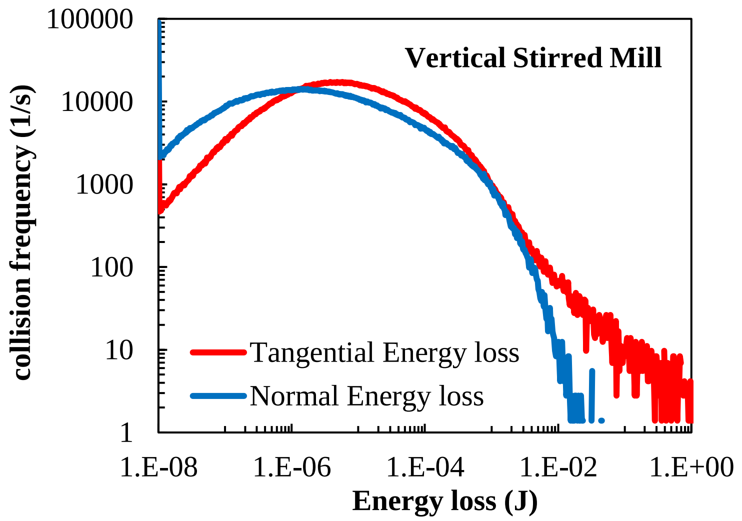

Figure 5. Energy and frequency of collisions for vertical stirred mill

Figure 6 compares the total energy loss, combining both the normal and the shear components, and the frequency of collisions for the two mills. It is observed that the energy spectrum for the vertical stirred mill has, approximately, a 10^{-2} J offset when compared to the curve that represents the ball mill. Also, the curves for the two mills show generally similar trends.

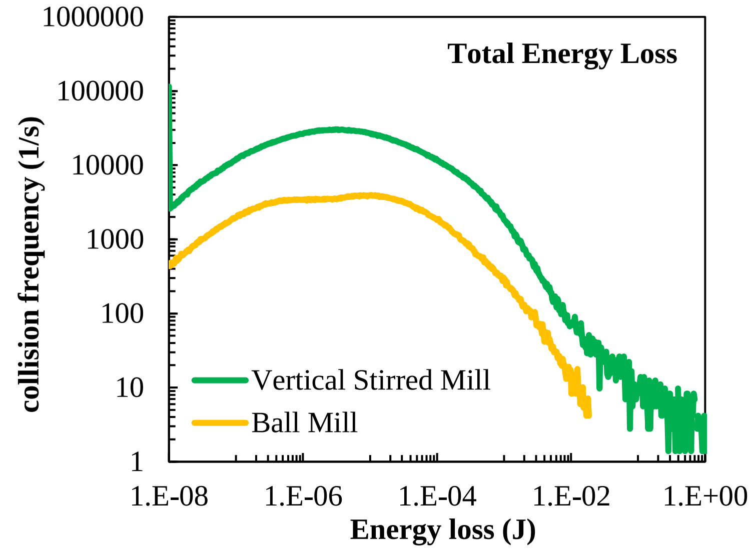

Figure 6. Comparison of the energies involved and frequency of collisions studied mills.

The collision energy spectra, presented in Figure 6, were used as input to the UFRJ mechanistic modeling approach to simulate the jar test. Figure 7 shows the fines generation (% < 53 microns) versus specific energy (kWh/t) obtained from UFRJ mechanistic model and experimental results for jar test, pilot test and industrial operation. Energy consumption for the jar test is already multiplied by 0.65 factor often used by Metso to scale-up Vertimill^{TM}.

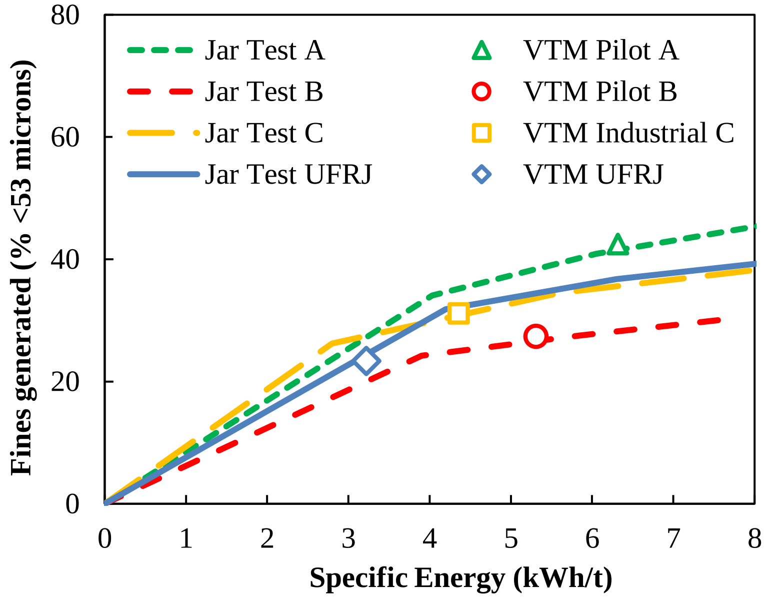

Figure 7. Fines generation as function of specific energy consumption

The lines represent the jar test fines generation from different grinding times and the symbols represent the pilot tests or industrial operation. The UFRJ mechanistic model results matches well as the experimental results does.

5. Conclusions

The energy spectra of mill charge of vertical stirred mill and ball mill (jar test) were obtained using the discrete element method and the results has confirmed the efficiency of vertical stirred mill. The simulated results represented the general outcome observed in practice. Predictions using the UFRJ mechanistic mill model demonstrated that the breakage rates are, indeed, higher for the vertical stirred mill under equivalent conditions to the jar test and that the scale-up methodology often used to the full-scale vertical stirred mill is valid.

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