

New constitutive equation for salt rock creep

Novo modelo constitutivo para fluência de rochas evaporíticas

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Abstract

Drilling and stabilizing oil wells are important technological challenges for developing the Brazilian Pre-Salt deposits due to creep behaviour of salt rocks, which results in well closure and collapse. The objectives of this work are to analyse the currently used creep models and to propose a new empirical model that comprehends the three creep stages. This work suggests that a deep interpretation of Burger's rheological model, based on the physical parameters of creep phenomenon, should be investigated. The problem of the explicit presence of time in creep strain rate models is discussed. The new proposed model is able to reproduce models that describe the transient and the steady-state creep stages. However, its calibration should be careful, based on data from all these creep stages.

Keywords: salt rock, oil drilling, creep models, rock mechanics.

Resumo

A perfuração e a estabilidade dos poços são importantes desafios tecnológicos para o desenvolvimento da província petrolífera do Pré-Sal brasileiro, graças à fluência das rochas evaporíticas, que provoca o fechamento e o colapso dos poços de pesquisa e de produção. Os objetivos desse trabalho são a análise dos modelos de fluência atualmente utilizados e a proposta de um novo modelo empírico, que compreenda todas as fases da fluência. É proposto que uma melhor interpretação do modelo reológico de Burger, baseada nos mecanismos físicos que regem a fluência, seja investigada. Uma discussão sobre o problema da presença explícita do tempo na formulação da taxa de deformação por fluência é revista. O novo modelo proposto é capaz de replicar modelos que representam a fluência primária e secundária, porém sua calibração deve ser criteriosa, com dados de todas essas fases da fluência.

Palavras Chave: evaporitos, poços de petróleo, fluência, mecânica de rochas.

1. Introduction

The Brazilian Pre-Salt is the biggest oil discovery in last fifty years. Made public in 2007, it could include Brazil among the biggest oil producers in the world. The reservoir is in a 3-4 km thick porous rock layer, under 1.5-3.0 km of seawater and a salt rock layer with up to 2.5 km of thickness (Riccomini *et al.*, 2012).

The drilling and stability of oil wells are some of the most important technological challenges for the Brazilian

Pre-Salt development. Creep behaviour of salt rocks causes serious issues to the drilling and stabilizing of oil wells. Closure and collapse due to wall convergence are constantly reported (Urai *et al.*, 2008; Mackay *et al.*, 2007; Ferro *et al.*, 2009).

Creep behaviour has been largely studied for the development of potash mines (Pinto, 1995), oil and gas well drilling (Mackay *et al.*, 2007; Poiate *et al.*, 2006) and nuclear waste disposal (Yao, 2007). Many constitutive equa-

tions, of empiric or rheological nature, have been proposed to describe the creep time-dependent strain (Pinto, 1995; Jaeger *et al.*, 2007; Grainger, 2012) but all of them have their limitations regarding the physical meaning of the phenomenon.

In this paper, the physical and mathematical hypotheses of salt rock' constitutive models are investigated and a new empirical model, which comprehends the whole phenomenon, is proposed.

Creep behaviour

The ideal time dependent strain behaviour was proposed by Jaeger (2007) and is shown in figure 1.

Upon the application of a stress, an evaporite undergoes transformation to

through an instantaneous elastic strain (A). The strain continues to increase with a decreasing rate in zone I, defined as primary or transient creep. The strain rate goes down to a minimum value,

which holds constant in zone II, defined as secondary or steady-state creep. Zone III represents a theoretical tertiary phase of creep, in which the strain rate increases until material failure (Cruz, 2003).

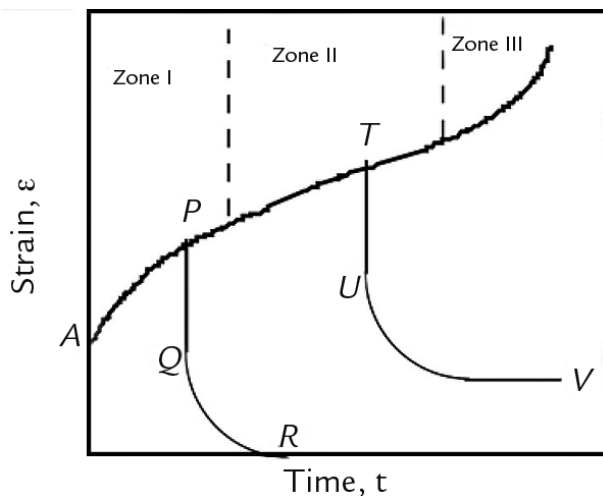


Figure 1
Theoretical curve of creep for constant deviatoric stress (adapted from Cruz, 2003)

In stress relief, the behaviour of material should follow path PRQ in zone I, without residual strain, and

path TUV in zone II, where there will be some residual strain (Jaeger *et al.*, 2007).

An adequate equation to describe the curve in Figure 1, as a function of the time, would be described as (1):

$$\epsilon = \epsilon_e + \epsilon_1(t) + \epsilon_2(t) + \epsilon_3(t) \tag{1}$$

Here ϵ_e represents the elastic strain and $\epsilon_1(t)$, $\epsilon_2(t)$ and $\epsilon_3(t)$ represents tran-

sient, steady-state and tertiary creep, respectively. The strain rate in each zone

would be:

$$\frac{\partial^2 \epsilon_1(t)}{\partial t^2} < 0 \tag{2}$$

$$\frac{\partial^2 \epsilon_2(t)}{\partial t^2} = 0 \tag{3}$$

$$\frac{\partial^2 \epsilon_3(t)}{\partial t^2} > 0 \tag{4}$$

Therefore, satisfying the conditions in equations (2), (3) and (4), any function could represent this model, providing a smooth transition from one creep stage to

another. These considerations imply that the total strain is the sum of elastic and creep strain. Experimental results show that the main variables that influence

creep behaviour are temperature (T) and deviatoric stress (σ^{dev}), not the hydrostatic stress (Jaeger *et al.*, 2007; Grainger, 2012). The total strain would be given by:

$$\epsilon = \frac{\sigma^{dev}}{2G} + \epsilon_{CR}(\sigma^{dev}, T, t) \tag{5}$$

This paper will classify the creep

models in two categories: the empirical

and the rheological ones.

Empirical models

The empirical models aim to fit the proposed functions to experimental data based or not in physical assumptions about the phenomena

mechanisms.

The most used empirical model for modelling rock creep is the Norton Power Law, proposed in 1929 (Pinto, 1995;

Yao, 2007; Cruz, 2003). It represents a constant strain rate, characteristic of the secondary creep, as shown in the following equation:

$$\dot{\epsilon}_{CR} = A \sigma^n \tag{6}$$

Here A and n are material parameters and σ is the deviatoric stress. The model is not capable of representing the primary creep (Pinto, 1995).

The temperature term considers the Arrhenius Law (Yao, 2007), Q represents the activation energy, R is the universal gas constant and T_0 is a temperature of reference. The term ϵ_0 represents a material parameter and σ_0 is a stress

where m is a new hardening parameter that should be smaller than zero (Pinto, 1995). However, the Bayle-Norton cannot provide a smooth transition to

Rheological models

One-dimensional rheological models define the materials as idealized ele-

This differential equation is able to reproduce primary and secondary creeps,

The model presents the instantaneous, time independent, elastic strain term, the second term that represents an exponential decreasing strain rate, representing primary creep, and a third linear, time dependent term, which represents

Presence of time parameter in creep equations

For all creep models, the strain is an explicit function of the time. The strain rate, according to the ideal model in Fig. 1, varies along the time. Physically, however, the variations should not

where ϵ_{dev} is the deviatoric strain and $\alpha < 0$ is a parameter of the mate-

In equation (12), A , n , β and γ

The Double Mechanism Law, developed with data from Taquari-Vassouras basin in Northeast of Brazil, has the same form of the Norton Power

$$\dot{\epsilon}_{CR} = \left[\left(\frac{\epsilon_0}{\sigma_0^n} \right) e^{\frac{Q}{RT_0} - \frac{Q}{RT}} \right] \sigma_{dev}^n \quad (7)$$

threshold: above σ_0 or below σ_0 different values of n are used, according to different creep mechanisms. This equation is useful for complex modelling and where changes in temperature are significant. Nevertheless, it fails to represent a truly

$$\dot{\epsilon}_{CR} = A \sigma^n t^m \quad (8)$$

secondary creep and the strain rate is infinite for $t = 0$. Thus, it is necessary to fix a positive initial time. The equation also shows that for increasing values of

ments, as springs and dashpots (Grainger, 2012). The most complete rheological

$$\eta_1 \ddot{\epsilon} + k_1 \dot{\epsilon} = \frac{\eta_1}{k_2} \ddot{\sigma} + \left(1 + \frac{k_1}{k_2} + \frac{\eta_1}{\eta_2} \right) \dot{\sigma} + \frac{k_1}{\eta_2} \sigma \quad (9)$$

according to the equation (6) solved for constant stress:

$$\epsilon(t) = \frac{\sigma_0}{k_2} + \frac{\sigma_0}{k_1} \left[1 - e^{-\frac{k_1}{\eta_1} t} \right] + \frac{\sigma_0}{\eta_2} t \quad (10)$$

secondary creep.

Rheological models are not used often in salt creep problems because they are not directly related to the creep mechanisms. However, they can provide mathematical background to the physical

be due to the time itself, but to changes in the characteristics of the material and in the environment. It can make it impossible to be used in every day models where different parts are submitted to

$$\dot{\epsilon}_{CR} = A \sigma^n \epsilon_{dev}^\alpha \quad (11)$$

rial. This model has a mathematical singularity for zero deviatoric strain.

$$\dot{\epsilon}_{CR} = \frac{A \sigma_{dev}^n}{(\gamma + \epsilon_{dev})^\beta} \quad (12)$$

are material parameters, ϵ_{dev} and σ_{dev} are deviatoric strain and stress, respec-

Law, with a term that considers temperature effect and a stress threshold (σ_0) (Grainger, 2012):

transient creep.

The Bayle-Norton Law, or Time Hardening Law, uses a time dependent term to represent the decreasing strain rate in primary creep, as shown in the following equation:

time, the Bayle-Norton Law tends to a horizontal straight line that represents steady-state creep instead of presenting a positive inclination.

model is the Burger's substance, whose equation is:

models. In addition, only the rheological models provide terms (time differentials of stress and strain) that consider stress and strain variations. The constants of Burger's model should be interpreted in terms of rock parameters.

deviatoric stresses at different moments. Some attempts to avoid the explicit use of time have been made, like the Strain Hardening Law (Pinto, 1995; Yao, 2007) [6, 7]:

Pinto (1995) proposed a new model that solved the problem:

tively. The equation would guarantee a smooth transition from primary creep

($\epsilon_{dev} \gg \gamma$) to secondary creep ($\epsilon_{dev} \ll \gamma$) (Pinto, 1995). However, during primary

creep, small strains occurs, characterizing model similar to the Norton Power Law:

$$\lim_{\epsilon \rightarrow 0} \frac{A \sigma_{dev}^n}{(\gamma + \epsilon_{dev})^\beta} = A' \sigma_{dev}^n \rightarrow A' = \frac{A}{\gamma^\beta} \quad (13)$$

For higher strain values, the model turns into a horizontal asymptote:

$$\lim_{\epsilon \rightarrow \infty} \frac{A \sigma_{dev}^n}{(\gamma + \epsilon_{dev})^\beta} = 0 \quad (14)$$

Therefore, the equation proposed by Pinto (1995) represents a truly time

hardening material, achieving finite strain rates for small strain values. The equation

depends only in material parameters, without explicit time dependence.

Proposing a new creep constitutive equation

A new empirical model would have two objectives: comprehend the three creep stages and eliminate any explicit function of the time in the strain rate

formulations. Following the structure of equation (1), the equation proposed by Pinto (1995) could be used as a primary creep term and the Double Mechanism

Law could be used as a secondary creep term. The tertiary creep would be an increasing function of the deviatoric strain. One resulting equation could be:

$$\dot{\epsilon}_{cr} = \frac{A_1 \sigma_{dev}^{n_1}}{(\gamma + \epsilon_{dev})^\beta} + \left[\left(\frac{\epsilon_0}{\sigma_0^{n_2}} \right) e^{\frac{Q}{RT_0} - \frac{Q}{RT}} \right] \sigma_{dev}^{n_2} + A_3 \sigma_{dev}^{n_3} \epsilon_{dev}^{n_3} \quad (15)$$

Due to the large number of parameters, some arbitrary simplifications will be made. First, $n_1 = n_2 = n_3 = n$, which means that the stress will have the same

influence in all creep stages. Then, the temperature will be considered as well to have the same influence in all three creep stages. Due to lack of data, the

stress threshold of the Double Mechanism Law will not be considered as well. Thus, the new constitutive model will be expressed as:

$$\dot{\epsilon}_{cr} = \left[\frac{A_1}{(\gamma + \epsilon_{dev})^\beta} + A_2 + A_3 \epsilon_{dev}^n \right] \sigma_{dev}^n e^{\frac{Q}{RT_0} - \frac{Q}{RT}} \quad (16)$$

The equation have seven material parameters ($A_1, A_2, A_3, \gamma, \beta, \alpha$ and n).

Q is the activation energy, R is the universal constant of the gases, T_0 is a

reference absolute temperature and T is the absolute temperature of the system.

ϵ_{dev} and σ_{dev} are the deviatoric strain and stress, respectively.

The equation should be calibrated with data from all three creep stages of the material are necessary.

Two-dimensional modelling

The equation was tested with the rock mechanics software FLAC 2D, from ITASCA, which utilizes a FDM/FVM (Finite Differences/Finite Volumes Method) algorithm (Cruz, 2003).

The software has a programming tool, FISH language, which allows the user to implement new constitutive models. More information about it can be found in Pinto (1995) and in FLAC

user's manual (ITASCA, 2006).

This work considers the von Mises deviatoric stress and strain as a measure of the magnitude of the distortion stresses.

2. Methodology

The element grid utilized in the tests is shown in figure 2.

The grid's intent is to simulate a well closure due to convergence of the wall.

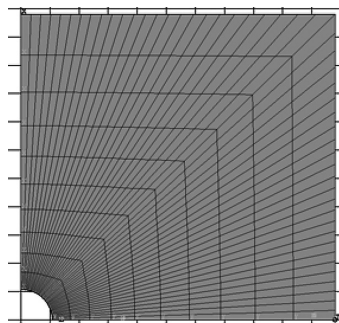


Figure 2
Element grid used in the test of the new creep constitutive model

The well radius was considered to be 0.108 m, according to Mackay (2007). The model extends until 1.08 m in order to minimize the excavation ends effects in the stresses and strain on the external border of the model. The grid has 10 cells in the radial direction and 50 cells in the tangential direction. Each cell length in the radial direction is 10% bigger than its predecessor.

The objective of the tests is to verify the ability of the new model to adhere to different models considering different materials.

The regions $x = 0$ and $y = 0$ were considered symmetry borders (no dis-

placement allowed in the perpendicular direction) and a horizontal stress of 47 MPa (compressive stress) was applied in the external borders ($x = 1.08$ and $y = 1.08$). This stress considered a 2,000 m thick water layer ($\rho = 1,000 \text{ kg/m}^3$) and a 1,000 m thick sediment layer ($\rho = 2,700 \text{ kg/m}^3$), conditions at the top of the salt layer in Brazilian Pre-Salt (Poiate *et al.*, 2006). The initial vertical stress was set to 47 MPa as well, as the evaporite materials only attain equilibrium in hydrostatic stress conditions (Pinto, 1995).

The tests considered two different materials: halite and taquidrite, evaporites present in the salt layer of

Santos and Campos basins (Mackay *et al.*, 2007).

The simulations were performed in three steps: first, the border of the well was fixed and the *in situ* stress conditions were calculated. Then, the border of the well was liberated and a new stage of equilibrium was attained without creep influence, which means that the elastic equilibrium was considered to be instantaneous. The last step was the creep displacement calculations stage, until the end time of 3014 days, approximately 8.5 years.

The parameters used in the tests are shown in the tables 1, 2 and 3.

Table 1
Physical parameters
of the materials used in the simulations

Materials	K (Mpa)	G (Mpa)	ρ (kg/m ³)
Halite	2.43x10 ⁴	7.50x10 ³	2.2x10 ³
Taquidrite	4.82x10 ³	1.85x10 ³	1.67x10 ³

Table 2
Parameters for the
simulations comparing
the new model to Pinto's model

Param.	Halite		Taquidrite	
	Pinto (1995)	Motta (2014)	Pinto (1995)	Motta (2014)
A ₁	1.5x10 ⁻²¹	1.5x10 ⁻²¹	6.5x10 ⁻¹⁵	6.3x10 ⁻¹⁵
A ₂	-	9.0x10 ⁻¹⁴	-	1.0x10 ⁻¹³
A ₃	-	1.0x10 ⁻¹⁵	-	1.0x10 ⁻¹⁷
n	1.8	1.8	1.9	1.9
α	-	2.0	-	3.0
β	9.75	9.75	3.0	3.0
γ	0.12	0.12	0.12	0.12

Table 3
Parameters
for the simulations comparing
the new model to Norton Power Law

Parameter	Halite	
	Norton Law	Motta (2013)
A ₁	-	1.0x10 ⁻³⁰
A ₂	5.0x10 ⁻¹³	5.0x10 ⁻¹³
A ₃	-	1.0x10 ⁻¹⁸
n	1.4	1.4
α	-	2.0
β	-	1.0
γ	-	1.0

The same density and the elastic parameters, shear modulus (G) and bulk modulus (K), were used in all tests. The parameters for Pinto's model and Norton Power Law were obtained from Cruz (2003) and consisted of

data from Taquari-Vassouras basin, in the Northeast of Brazil. The parameters of the new model were defined in order to attain as close as possible to the correspondent parameters in the other models.

As the new model has more parameters than the others do, these parameters were set to have small values. When the linear (A_i) parameters have small values, the power (Greek letters) do not have considerable influence.

3. Results

Figure 3 shows radial displacement of the well border as function of the creep time for the two materials.

The plotted data from different models show completely different behaviours:

a decreasing rate for Pinto's model and a linear increase for Norton Power Law.

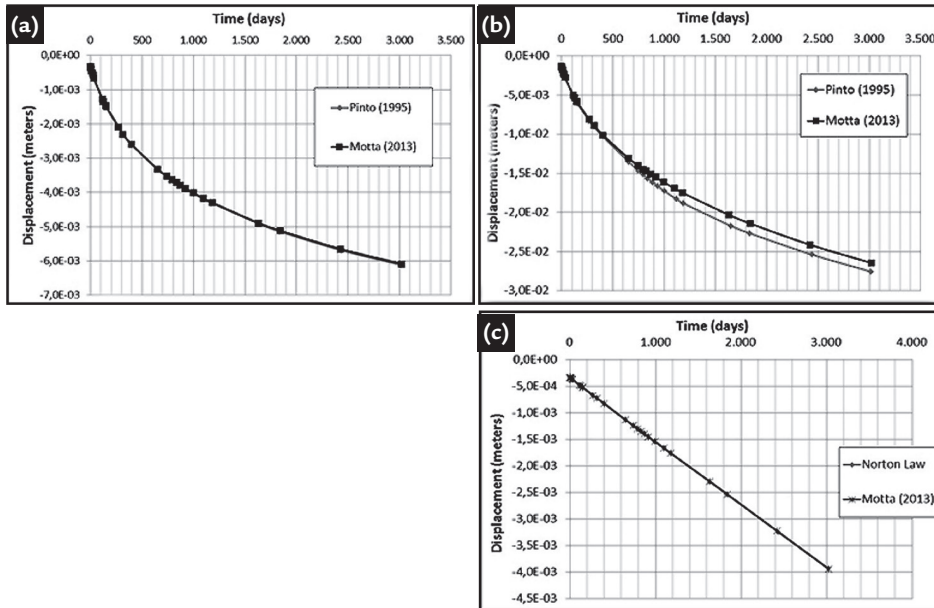


Figure 3
Displacement versus creep time: comparison between (a) the new model and Pinto's model with halite; (b) the new model and Pinto's model with taquidrite; (c) the new model and Norton Power Law with halite

The results show that the new model showed a good adherence to the two different models that represent two different creep stages. The closer the models' parameters are, the closer the curves. A difference of 0.2 in the A1 parameter for the taquidrite test resulted in a slight difference between the curves.

The additional parameters of the new model, set up as minimum, did not influenced the result; these parameters could be excluded when the creep stage that they do represent are not of interest: the model could be reduced to Pinto's model with no loss.

Figure 4 shows the deviatoric stress

in the border of the well versus the creep displacement.

The new model reproduces the stress behaviour defined by both Pinto's model and Norton Power Law. The difference observed in Pinto's model comparison can be assigned to a restricted increase in the unbalanced force.

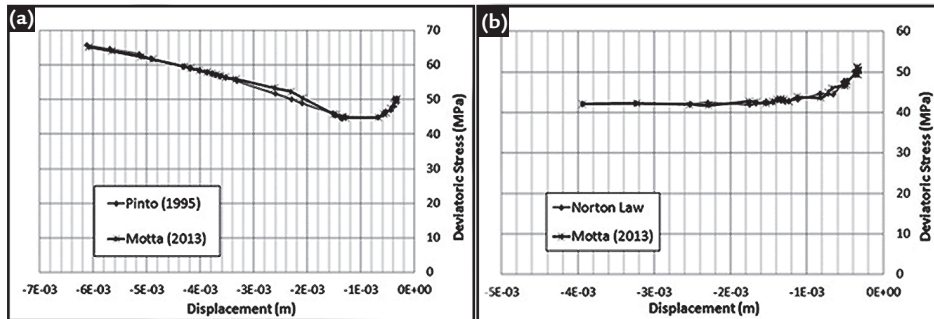


Figure 4
Deviatoric stress at the well borders (with halite): (a) comparison between the new model and Pinto's model; (b) comparison between the new model and Norton Power Law

It is interesting to observe that none of the models provides a systematic decrease in the deviatoric stress due to strain. As creep strain occurs only at the presence of a deviatoric stress, it should provide a stress relief on the rock mass (Pinto, 1995). Pinto's model returns an increasing deviatoric stress after a minimum value

and Norton Power Law shows a static deviatoric stress for high time intervals. In both cases, creep strain would occur indefinitely. The new model will not solve this problem, as it reproduces exactly the same stress behaviour as the others. Therefore, none of the models should be used for extrapolation.

$$A' = \frac{A_1}{(\gamma + \epsilon_{dev})^\beta} \tag{17}$$

The defined creep index (A') would reduce as the deviatoric stress increases, characterizing the primary creep. If the creep index was a function of time, its value would be constant for every region of the model, as the time passes equally in every point of the model. Figure 5 shows

that A' value is different in different regions of the model, as the material suffer different influences from the excavation.

It is possible to see that the more distant from the centre of the excavation, the higher is its creep index. This can be assigned to the lower deviatoric strain ob-

In Pinto's model, the strain rate is a function of deviatoric strain and deviatoric stress. The deviatoric strain is responsible for determining the influence of the material in the strain rate: the more the material is distorted, the higher is the rate of distortion. One can define a 'creep index' according to:

served at distant points. If the creep index was a function of time (as in Bayle-Norton Law, for example) the three curves would superpose each other. It is worth mentioning that the horizontal asymptote $\gamma = 0$ observed in the 0.14m curve, agrees with the mathematical analysis.

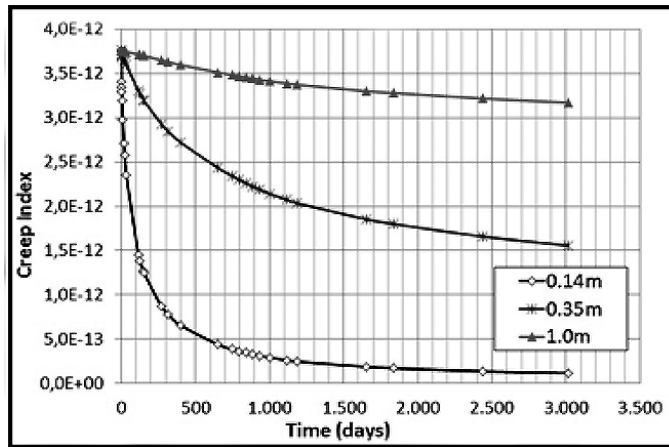


Figure 5
Creep index (A') behaviour:
creep strain rate decrease with time

4. Conclusions

The analysis of the empirical models showed that none of the ones proposed in the literature comprehends more than one creep stage, and some of them have mathematical problems with small time and strain values. Empirical equations should not be used to extrapolation problems. This work suggests a deeper interpretation of the physical

meaning of the Burger's model, in order to solve the mathematical problems of the empirical models.

The discussion about the problem of the strain rate as an explicit function of time was demonstrated by the results, which showed different creep strain rates in different regions of the model. As the time passes equally all around the model,

it should not directly define the strain rate of the material.

The new model was able to reproduce the behaviour of two different creep models that represent two distinct creep stages. However, the new equation requires a careful calibration due the larger number of parameters. Data from all creep stages are needed.

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